Binomial Theorem (Part 1)

These notes are intended as a summary of section 8.6 (p. 738 – 742) in your workbook. You should also read the section for more complete explanations and additional examples.

Expand and simplify each of the following binomials:

$(x+y)^0 =$

$(x+y)^1 =$

$(x+y)^2 =$

$(x+y)^3 =$

$(x+y)^4 =$

$(x+y)^5 =$

This becomes tedious very quickly. Fortunately, by observing some patterns, the process can be greatly simplified.

Observations

1. In each term of the expansion, the sum of the exponents is equal to the power of the binomial.

2. The exponent of the first term of the binomial (x) in each term of the expansion decreases from left to right.

3. The exponent of the second term of the binomial (y) in each term of the expansion increases from left to right.

4. The number of terms in the expansion is one more than the power of the binomial.

Note:

- If the binomial is of the form $(x+y)^n$, all terms in the expansion will be positive.
- If the binomial is of the form $(x+y)^n$, the terms in the expansion will alternate +, –, +, –, etc.

Based on these observations, any expansion of a binomial will have the form:

$$(x+y)^n = \_\_x^n + \_\_x^{n-1}y + \_\_x^{n-2}y^2 + \ldots + \_\_x^2y^{n-2} + \_\_xy^{n-1} + \_\_y^n$$

with coefficients in the blanks.

Example (not in workbook)

Write the expansion of $(x+y)^6$, leaving the coefficients blank for now.

There are two methods that can be used to determine the coefficients.

Method 1 — Pascal's Triangle

Pascal's triangle is developed by listing the coefficient of the expansions of $(x+y)^n$, for increasing values of n, in a triangular array.

n	$(x+y)^n$	Coefficients
0	$(x+y)^0$	1
1	$(x+y)^1$	1 1
2	$(x+y)^2$	1 2 1
3	$(x+y)^3$	1 3 3 1
4	$(x+y)^4$	1 4 6 4 1
5	$(x+y)^5$	1 5 10 10 5 1
6	$(x+y)^6$	
7	$(x+y)^7$	

Note the following pattern:

1. The first and last number in each row are always 1.

2. The coefficients in each row are the same read forwards or backwards.

3. Beginning with the second row, the sum of two adjacent entries is equal to the entry "between them" in the next row.

Example (not in workbook)

Use Pascal's triangle to write the expansion of $(x+y)^6$ with coefficients.

The only problem with Pascal's triangle is that it becomes difficult (and time consuming) to develop for large values of n.

Example 1 (sidebar p. 739)

Expand, then simplify $(3b-1)^4$.

Example 2 (sidebar p. 741)

Expand and simplify $(4a^2+2b)^3$.

Homework: #3, 4, 8 in the section 8.6 exercises (p. 743 – 749). Answers on p. 750.